Section 1.2 Limits part 2 (Minimum homework: 1 – 29 odds, 33, 35, 37)

We are asked to use Algebra to find limits of functions in section 1.2.

The first step to attempt to find the limits in section should be to plug the given value into the function.

- The result is the limit if the answer is a real number.
- The limit may or may not exist if the answer is not a real number. Further work is required.
    - If the Algebra produces the fraction $\frac{0}{0}$ the limit likely exists
    - If the Algebra produces the fraction $\frac{a}{0}$ $where\ a\ is\ some\ real\ number\ other\ than\ 0$ The limit does not exist

Example 1: Find the following limit:

$\lim_{x\to 5}(3x-9)$

Step 1: Plug in $x=5$

$\lim_{x\to 5} 3x-9 = 3(5)-9 = 15-9 = 6$

The result is a well-defined real number, this is the limit.

Answer: $\lim_{x\to 5}(3x-9)$ =6

Example 2: Find the following limit:

$\lim_{x\to 1} \frac{x-1}{x^2+x-2}$

Step 1: Plug in $x = 1$

$\lim_{x\to 1} \frac{x-1}{x^2+x-2} = \frac{1-1}{1^2+1-2} = \frac{0}{0}$

When step 1 produces the fraction $\frac{0}{0}$, the limit likely exists. Algebra will be needed to find the limit.

In this case the Algebra is to:

- factor,
- cancel
- plug in $x = 1$

$$\lim_{x\to 1} \frac{x-1}{x^2+x-2} = \lim_{x\to 1} \frac{(x-1)}{(x-1)(x+2)}$$

$$= \lim_{x\to 1} \frac{1}{x+2}$$

$$= \frac{1}{1+2}$$

$$= \frac{1}{3}$$

Example 3: Find the following limit.

$$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$$

Step 1: Plug in $x = 4$

$$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \frac{\sqrt{4}-2}{4-4} = \frac{2-2}{0} = \frac{0}{0}$$

When step 1 produces the fraction $\frac{0}{0}$, the limit likely exists. Algebra will be needed to find the limit.

In this case the Algebra is to:

- Multiply by the conjugate of the numerator
- FOIL / simplify the numerator
- Do NOT FOIL / simplify the denominator
- Cancel
- plug in $x = 4$

For this problem, you multiply the numerator and denominator by the conjugate of $\sqrt{x}-2$ which is $\sqrt{x}+2$

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$$

$$=\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}*\frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$=\lim_{x\to 4}\frac{x+2\sqrt{x}-2\sqrt{x}-4}{(x-4)(\sqrt{x}+2)}$$

$$=\lim_{x\to 4}\frac{x-4}{(x-4)(\sqrt{x}+2)}$$

$$=\lim_{x\to 4}\frac{1}{\sqrt{x}+2}$$

$$=\frac{1}{\sqrt{4}+2}$$

$$=\frac{1}{4}$$

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}=1/4$$

Example 4: Find the following limit at infinity.

$$\lim_{x\to\infty} \frac{2x+1}{5x+4}$$

Step one: plug in $x = \infty$

$$\lim_{x\to\infty} \frac{2x+1}{5x+4} = \frac{2(\infty)+1}{5(\infty)+4} = \frac{\infty}{\infty}$$

The limit may exist. This step didn't help

Multiply the fraction by

$$\frac{1}{x} over \frac{1}{x} \quad (the\ highest\ power\ of\ x\ in\ the\ problem)$$

$$\lim_{x\to\infty} \frac{2x+1}{5x+4} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x}}\left(\frac{2x+1}{5x+4}\right)$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}*2x+\frac{1}{x}*1}{\frac{1}{x}*5x+\frac{1}{x}*4}$$

$$= \lim_{x\to\infty} \frac{2+\frac{1}{x}}{5+4/x}$$

$$= \frac{2+\frac{1}{\infty}}{5+\frac{4}{\infty}}$$

(fractions with a real number in the numerator and infinity in the denominator are equivalent to 0)

$$\cong \frac{2+0}{5+0}$$

$$= 2/5$$

$$\lim_{x\to\infty} \frac{2x+1}{5x+4} = \frac{2}{5}$$

Example 5: Find the following limit:

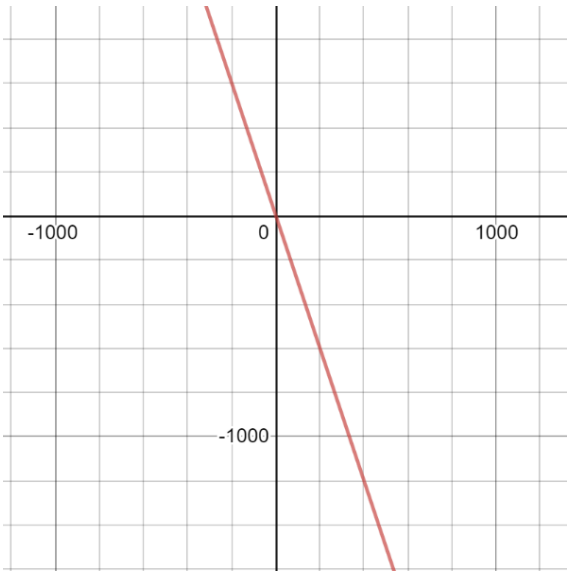
$$\lim_{x\to\infty}(-3x+4)$$

The limit of a polynomial as x approaches infinity will equal the limit as the term with highest degree in the polynomial.

That is $\lim_{x\to\infty}(-3x+4) = \lim_{x\to\infty}(-3x)$

- This limit can only be positive infinity (∞) or negative infinity $(-\infty)$.
- We need to use logic or sketch a graph to figure out which it is.
- I will try to explain the logic approach in a video. For now let us find this limit graphically.

$\lim_{x\to\infty}(-3x) = -\infty$ from the graph.



Answer: $\lim_{x\to\infty}(-3x+4) = -\infty$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

#1-20: Find the following limits using Algebra.

1) $\lim_{x \to 5}(2x + 6)$

2) $\lim_{x \to -3}(5x - 7)$

Answer: $\lim_{x \to -3}(5x - 7) = -22$

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3) $\lim_{x \to 3}(x^2 + 5x - 4)$

4) $\lim_{x \to 2}(x^2 - 3x - 7)$

Answer: $\lim_{x \to 2}(x^2 - 3x - 7) = -9$

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5) $\lim_{x \to 4} \sqrt{x + 5}$

6) $\lim_{x \to -3} \sqrt{2x + 10}$

Answer: $\lim_{x \to -3} \sqrt{2x + 10} = 2$

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7) $\lim_{x \to 2} \frac{3x+6}{x-5}$

8) $\lim_{x \to 4} \frac{2x+1}{x+7}$

Answer: $\lim_{x \to 4} \frac{2x+1}{x+7} = \frac{9}{11}$

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9) $\lim_{x \to -2} \frac{x^2+5x+6}{x^2+3x+2}$

10) $\lim_{x \to 1} \frac{x^2+4x-5}{x^2-1}$

Answer: $\lim_{x \to 1} \frac{x^2+4x-5}{x^2-1} = 3$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

11) $\lim_{x\to 3} \frac{x^2-9}{x^2-4x+3}$

12) $\lim_{x\to -2} \frac{x^2-4}{x^2-3x+2}$

Answer: $\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2} = 0$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

13) $\lim_{x\to 2} \frac{2x^2-3x-2}{3x^2-2x-8}$

14) $\lim_{x\to 3} \frac{2x^2-7x+3}{3x^2-7x-6}$

Answer: $\lim_{x\to 3} \frac{2x^2-7x+3}{3x^2-7x-6} = \frac{5}{11}$ $(did\ this\ in\ my\ head, we\ shall\ see\ if\ this\ is\ correct)$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

15) $\lim_{x \to 16} \frac{\sqrt{x}-4}{x-16}$

16) $\lim_{x \to 49} \frac{\sqrt{x}-7}{x-49}$

Answer: $\lim_{x \to 49} \frac{\sqrt{x}-7}{x-49} = \frac{1}{14}$

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17) $\lim_{x\to 36} \frac{\sqrt{x}-6}{x-36}$

18) $\lim_{x\to 64} \frac{\sqrt{x}-8}{x-64}$

Answer: $\lim_{x\to 64} \frac{\sqrt{x}-8}{x-64} = \frac{1}{16}$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

19) $\lim_{x \to 121} \frac{\sqrt{x}-11}{x-121}$

20) $\lim_{x \to 25} \frac{\sqrt{x}-5}{x-25}$

Answer: $\lim_{x \to 25} \frac{\sqrt{x}-5}{x-25} = \frac{1}{10}$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

#21-32: Find the following limits using Algebra.

21) $\lim_{x\to\infty} \frac{3x+6}{2x-4}$

22) $\lim_{x\to\infty} \frac{3x+1}{5x-4}$

Answer: $\lim_{x\to\infty} \frac{3x+1}{5x-4} = \frac{3}{5}$

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23) $\lim_{x\to\infty} \frac{4x^2-3x+6}{5x^2+2x-4}$

24) $\lim_{x\to\infty} \frac{3x^2+2x-5}{7x^2+4x-2}$

Answer: $\lim_{x\to\infty} \frac{3x^2+2x-5}{7x^2+4x-2} = \frac{3}{7}$

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25) $\lim_{x\to\infty} \frac{3x+6}{2x^2-4}$

26) $\lim_{x\to\infty} \frac{3x+1}{5x^2-4}$

Answer: $\lim_{x\to\infty} \frac{3x+1}{5x^2-4} = 0$

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27) $\lim_{x\to\infty} \frac{4x^2-3x+6}{5x^3+2x-4}$

28) $\lim_{x\to\infty} \frac{3x^2+2x-5}{7x^3+4x-2}$

Answer: $\lim_{x\to\infty} \frac{3x^2+2x-5}{7x^3+4x-2} = 0$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

29) $\lim_{x\to\infty} \frac{3x^2+6}{2x-4}$

30) $\lim_{x\to\infty} \frac{3x^2+1}{5x-4}$

Answer: $\lim_{x\to\infty} \frac{3x^2+1}{5x-4} = \infty$

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31) $\lim_{x\to\infty} \frac{4x^3-3x+6}{5x^2+2x-4}$

32) $\lim_{x\to\infty} \frac{3x^3+2x-5}{7x^2+4x-2}$

Answer: $\lim_{x\to\infty} \frac{3x^3+2x-5}{7x^2+4x-2} = \infty$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

#33-40: Find the following limits using Algebra.

33) $\lim_{x\to\infty}(2x+6)$

34) $\lim_{x\to\infty}(5x-7)$

Answer: $\lim_{x\to\infty}(5x-7)\infty$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

35) $\lim_{x\to\infty}(-2x+6)$

36) $\lim_{x\to\infty}(-5x-7)$

Answer: $\lim_{x\to\infty}(-5x-7)=-\infty$

(Minimum homework: 1 – 29 odds, 33, 35, 37)

37) $\lim_{x\to\infty}(4x^2+5x-4)$

38) $\lim_{x\to\infty}(2x^2-3x-7)$

Answer: $\lim_{x\to\infty}(2x^2-3x-7)=\infty$

39) $\lim_{x\to\infty}(-4x^2+5x-4)$

40) $\lim_{x\to\infty}(-2x^2-3x-7)$